Manipulating Majorana fermions in one-dimensional spin-orbit coupled atomic Fermi gases

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Majorana fermions are promising candidates for storing and processing information in topological quantum computation. The ability to control such individual information carriers in trapped ultracold atomic Fermi gases is a novel theme in quantum information science. However, fermionic atoms are neutral and thus are difficult to manipulate. Here, we theoretically investigate the control of emergent Majorana fermions in one-dimensional spin-orbit coupled atomic Fermi gases. We discuss (i) how to move Majorana fermions by increasing or decreasing an effective Zeeman field, which acts like a solid state control voltage gate; and (ii) how to create a pair of Majorana fermions by adding a magnetic impurity potential. We discuss the experimental realization of our control scheme in an ultracold Fermi gas of ^{40}K atoms.

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Majorana fermions - particles that are their own antiparticles - were proposed by Ettore Majorana in 1937 [1]. They are finding ever wider relevance in modern physics [2]. A well-known proposal is the use of Majorana fermions as information carriers in topological quantum information processing [3, 4]. Due to the robustness of topological quantum computation, research into the creation and manipulation of Majorana fermions has become an active research topic in a variety of fields of physics, ranging from condensed matter physics to ultracold atomic physics. Most theoretical schemes of processing topological quantum information based on Majorana fermions have relied on using solid-state systems [5, 6]. Evidence of the existence of Majorana fermions in hybrid superconductor-semiconductor nanowire devices has been reported very recently [7].

The realization of Majorana fermions in ultracold atomic systems has also been discussed extensively [8–13]. Compared with solid state systems, a unique advantage of ultracold atoms is their unprecedented controllability and purity, which may be expected to lead to reduced decoherence. A vast range of interactions, geometries and dimensions are possible: using the tool of Feshbach resonances, one can control the interatomic interactions very accurately. By using the technique of optical lattices, one can create artificial one-dimensional (1D) or two-dimensional (2D) environments to explore how physics changes with dimensionality. In this context, topological quantum computation based on Majorana zero-energy modes in the vortex core of a 2D atomic Fermi gas has been discussed [8]. However, it is very difficult to experimentally generate and manipulate a vortex lattice in a fermionic superfluid.

In this Brief Report, we theoretically investigate the manipulation of Majorana fermions in one-dimensional spin-orbit coupled atomic Fermi gases. The existence of Majorana fermions in such systems was recently checked by us, by performing fully microscopic calculations within the mean-field Bogoliubov-de Gennes (BdG) theory [9]. Here, we study how to control Majorana fermions using a background magnetic field combined with a magnetic impurity potential. Our investigation is motived by the recent realization of a three-dimensional spin-orbit coupled Fermi gas of ^{40}K and ^{6}Li atoms, at ShanXi University [14] and at MIT [15], respectively. We anticipate that a 1D spin-orbit coupled Fermi gas will be created very soon, by using 2D optical lattices [16]. At the end of this paper, we discuss the possible experimental realization of our control scheme for ^{40}K atoms.

Model Hamiltonian. - Our theoretical approach is outlined in detail in the previous work [9]. Here we extend this approach to include a classical magnetic impurity. In the following, we introduce briefly the essential ingredients of the theory. The model Hamiltonian of a 1D atomic Fermi gas with spin-orbit coupling, as realized at ShanXi [14] and at MIT [15], can be written as

$$\begin{aligned}\mathcal{H} &= \int dx\psi^{\dagger}(x)\left[\mathcal{H}_{\sigma}^{S}(x)-h\sigma_z+\lambda k\sigma_y\right]\psi(x)\\&+g_{1D}\int dx\psi_{\uparrow}^{\dagger}(x)\psi_{\downarrow}^{\dagger}(x)\psi_{\downarrow}(x)\psi_{\uparrow}(x),\end{aligned}\qquad(1)$$

where $\psi^{\dagger}(x)\equiv[\psi_{\uparrow}^{\dagger}(x),\psi_{\downarrow}^{\dagger}(x)]$ denote collectively the creation field operators for spin-up and spin-down atoms, h denotes the strength of an effective magnetic field causing a Zeeman splitting, while $\lambda k\sigma_y\equiv -i\lambda(\partial/\partial x)\sigma_y$ is the spin-orbit coupling term with coupling strength λ. In addition, σ_y and σ_z are the usual 2×2 Pauli matrices while $g_{1D}=-2\hbar^2/(ma_{1D})$ is the effective Hamiltonian coupling given an s-wave scattering length in one dimension of a_{1D}. We note that the existence of spin-orbit coupling, magnetic fields and quantum transport to the cloud edge remove all of the usual symmetry properties of Fermi

gases, and these systems therefore provide an opportunity to investigate the least symmetric of the nonstandard symmetry classes in normal-superconducting hybrid structures[17].

The single-particle Hamiltonian $\mathcal{H}_\sigma^S(x)$ describes the atomic motion in a harmonic trapping potential $m\omega^2 x^2/2$, together with a scattering potential of a classical magnetic impurity, modeled as $-V_{imp}(x)\,\sigma_z$. Combining these terms, $\mathcal{H}_\sigma^S(x)$ takes the form,

$$\mathcal{H}_\sigma^S(x) = -(\hbar^2/2m)(\partial^2/\partial x^2) + (m/2)\omega^2 x^2 - V_{imp}(x)\sigma_z - \mu, \tag{2}$$

where μ is the chemical potential. The impurity scattering potential with strength V_0 is taken to have a gaussian shape, so that

$$V_{imp}(x) = [V_0/(\sqrt{2\pi}d)]\exp[-(x-x_0)^2/(2d^2)], \tag{3}$$

where x_0 and d are the position and width of the impurity potential, respectively. Thus, in the limit of infinitely narrow width where $d \to 0$, we obtain a Dirac delta-function like potential, $V_{imp}(x) = V_0\delta(x-x_0)$.

We solve the model Hamiltonian (1) using a mean-field Bogoliubov-de Gennes (BdG) approach. The wave-function of low-energy quasiparticles $[u_{\uparrow\eta}(x), u_{\downarrow\eta}(x), v_{\uparrow\eta}(x), v_{\downarrow\eta}(x)]^T$ with energy E_η satisfies the BdG equation,

$$\begin{bmatrix} \mathcal{H}_\uparrow^S(x)-h & -\lambda\partial/\partial x & 0 & -\Delta(x) \\ \lambda\partial/\partial x & \mathcal{H}_\downarrow^S(x)+h & \Delta(x) & 0 \\ 0 & \Delta^*(x) & -\mathcal{H}_\uparrow^S(x)+h & \lambda\partial/\partial x \\ -\Delta^*(x) & 0 & -\lambda\partial/\partial x & -\mathcal{H}_\downarrow^S(x)-h \end{bmatrix} \begin{bmatrix} u_{\uparrow\eta}(x) \\ u_{\downarrow\eta}(x) \\ v_{\uparrow\eta}(x) \\ v_{\downarrow\eta}(x) \end{bmatrix} = E_\eta \begin{bmatrix} u_{\uparrow\eta}(x) \\ u_{\downarrow\eta}(x) \\ v_{\uparrow\eta}(x) \\ v_{\downarrow\eta}(x) \end{bmatrix}, \tag{4}$$

where the pairing field is defined as:

$$\Delta(x) = -(g_{1D}/2)\sum_\eta [u_{\uparrow\eta}(x)v_{\downarrow\eta}^*(x)f(E_\eta) + u_{\downarrow\eta}(x)v_{\uparrow\eta}^*(x)f(-E_\eta)] \tag{5}$$

and the density of spin-σ atoms is given by $n_\sigma(x) = (1/2)\sum_\eta[|u_{\sigma\eta}(x)|^2 f(E_\eta) + |v_{\sigma\eta}(x)|^2 f(-E_\eta)]$. Here, $f(E) \equiv 1/[e^{E/(k_BT)}+1]$ is the Fermi distribution function at temperature T. The BdG equation (4) can be solved self-consistently using a hybrid method detailed in Ref. [18].

In the following calculations we consider a typical dimensionless interaction parameter of $\gamma \equiv (a_{ho}/a_{1D})/(\pi N^{1/2}) = 1.6$, where $a_{ho} \equiv \sqrt{\hbar/(m\omega)}$ is the characteristic oscillator length of the trap and $N = 100$ is the total number of fermionic atoms. For spin-orbit coupling, we take $\lambda k_F/E_F = 1$, where k_F and E_F are respectively the Fermi wave-vector and Fermi energy. The unit of length is the Thomas-Fermi radius, $x_F = N^{1/2}a_{ho}$. All calculations will be carried out at zero temperature, as the inclusion of a finite but small temperature will only marginally affect our results.

A brief summary of previous results. - In the *absence* of magnetic impurity, the spin-orbit coupled 1D Fermi gas in a harmonic trap will locally enter an interesting topological phase when the Zeeman field satisfies $h > h_c(x)$, as we reported earlier in Ref. [9]. Here $h_c(x) = \sqrt{\mu^2(x)+\Delta^2(x)}$ with $\mu(x) = \mu - m\omega^2x^2/2$ is the local critical field at position x. For interaction strength $\gamma = 1.6$ and spin-orbit coupling $\lambda k_F/E_F = 1$ mentioned above, this occurs at $h > h_c \sim 0.35E_F$ See, for example, the phase diagram in Fig. 1. At higher magnetic fields of $h > h_c$, a mixed phase emerges, consisting of a standard BCS superfluid at the trap center and a topological superfluid at the two edges of the trap (see Fig. 2).

As a salient feature of topological superfluids, Majorana fermions appear at the edges. These Majorana fermions are nothing but the zero-energy solutions of the BdG equation. Because of the particle-hole symmetry of the BdG equation, the field operator of Bogoliubov quasiparticles satisfies $\Gamma_E = \Gamma_{-E}^\dagger$. If the energy $E = 0$, we would then have $\Gamma_0 = \Gamma_0^\dagger$, which is the defining fea-

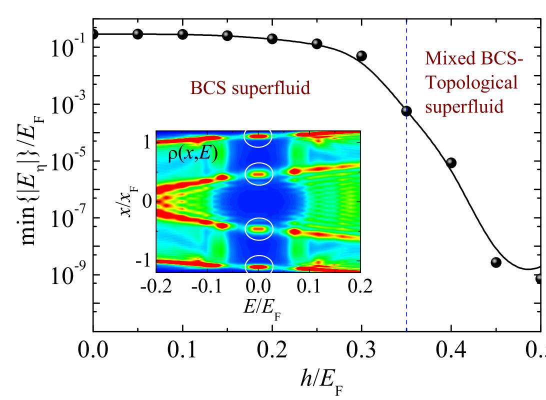

Figure 1: (color online) Phase diagram determined from the lowest energy of Bogoliubov quasiparticle spectrum. The inset shows the contour plot of local density of state at $h/E_F = 0.5$. The contributions from Majorana fermions are highlighted by circles.

ture of Majorana fermions. Related to this zero-energy feature, it is easy to see that spatially resolved radio-frequency (r.f.) spectroscopy, which measures the local density of state of the atoms, defined as $\rho(x,E) = (1/2)\sum_{\sigma\eta}[|u_{\sigma\eta}(x)|^2\,\delta(E-E_\eta)+|v_{\sigma\eta}(x)|^2\,\delta(E+E_\eta)]$, provides a useful means to identify Majorana fermions. In the inset of Fig. 1, we can identify from r.f. spectroscopy four zero-energy Majorana fermions at the Zeeman field $h = 0.5E_F$, two for each topological superfluid.

Manipulating Majorana fermions. - Let us consider how to manipulate Majorana fermions in 1D atomic Fermi gas. We focus on two operations: to move and to create Majorana fermions.

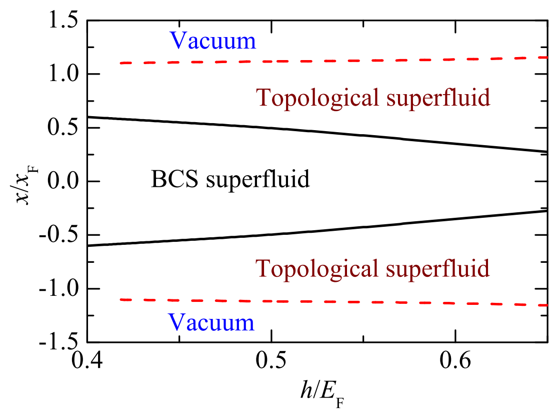

Figure 2: (color online) Positions of Majorana fermions as a function of Zeeman field.

We propose to use the Zeeman field, as an analog to a voltage gate in solid-state systems [6], to transport Majorana fermions through the whole Fermi cloud. Indeed, the position of Majorana fermions depends critically on the Zeeman field, as shown in Fig. 2, where we plot the boundary of each superfluid phase. The four Majorana fermions sit exactly on these boundaries. Therefore, by *adiabatically* increasing or decreasing the Zeeman field, the inner two Majorana fermions will move towards or away from the trap center. The position of the outer two Majorana fermions, however, is less affected by the Zeeman field.

To create new Majorana fermions in the system, we may consider either (i) creating a new topological superfluid in some areas of the Fermi cloud or (ii) introducing a strong defect in the existing topological superfluids. In the former case, a new pair of Majorana fermions will appear at the edges of the induced new topological superfluid. In the latter case, the strong defect will create a low-density hole, which is basically vacuum and thus topologically trivial. New Majorana fermions will then be trapped at the hole center. A well-known example of the latter case is a vortex in 2D topological superfluid, which hosts a zero-energy Majorana fermion mode inside the vortex core.

The main result of this work is that these two ideas (i) and (ii) of creating new Majorana fermions can be realized by introducing a classical magnetic impurity. The magnetic impurity serves as an attractive or repulsive scattering potential for spin-up or spin-down atoms, respectively, as we mentioned earlier in the model Hamiltonian. Due to the finite width of the impurity scattering potential, the first idea (i) seems more practical. In the following, we will focus on this idea and examine it by using self-consistent BdG calculations. To test this concept, we will put an impurity at the center of the Fermi cloud (i.e., $x_0 = 0$) and take a potential width $d = 0.07x_F$. The scattering strength of the impurity is taken as $V_0 = 0.13x_FE_F$.

The existence of impurity induced Majorana fermions is clearly evident from the local density of state, which is measurable using spatially resolved r.f. spectroscopy, as reported in Fig. 3a using a linear contour plot. For comparison, we show also in Fig. 3b the expected spectroscopy scan without an impurity. It is clear that a new pair of Majorana fermions is created at about $x = \pm 0.1x_F$, while other four Majorana fermions almost keep the same positions as those without the impurity potential. The creation of new Majorana fermions can be easily understood. The magnetic impurity potential with a relatively broader potential width may be regarded as a local Zeeman field. By tuning the strength of the magnetic impurity, we can greatly enhance the local Zeeman field at a particular area. This creates a local topological superfluid that hosts new Majorana fermions at its edges. To check this picture, we have calculated the criterion for a local topological superfluid, $h - h_c(x) > 0$. As highlighted in Fig. 4a by a pink cross-pattern, we find that this criterion is indeed satisfied at about the origin, where the magnetic impurity is located. The superfluid gap is significantly reduced at the magnetic impurity.

We finally check how the density distribution is affected by magnetic impurity. In Fig. 5a, we present the spin-up and spin-down density distributions and their difference $\Delta n(x) = n_\uparrow(x) - n_\downarrow(x)$. Compared with the result without the impurity (Fig. 5b), we find that the

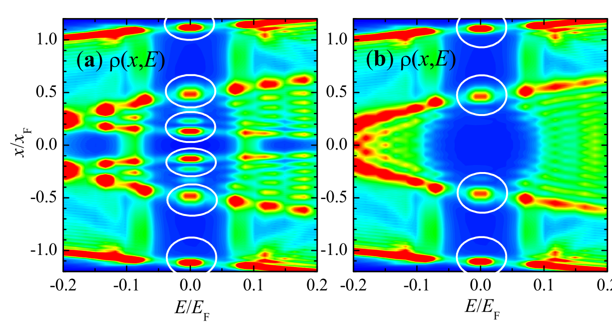

Figure 3: (color online) Linear contour plot of the local density of states with (a) or without (b) a magnetic impurity at $h/E_F = 0.5$.

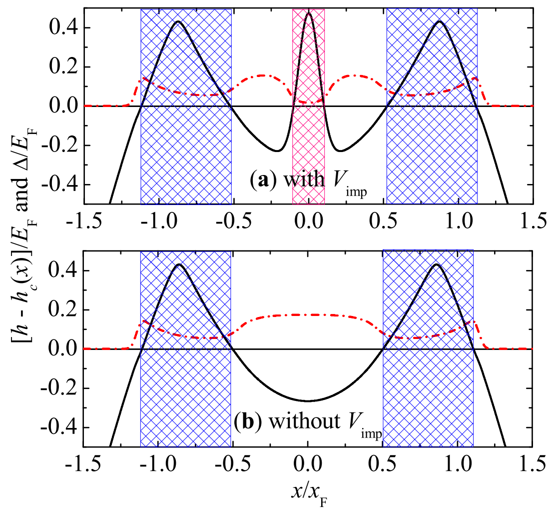

Figure 4: (color online) Criterion for topological superfluid $h - h_c(x)$ (solid lines) and the superfluid order parameter $\Delta(x)$ (dot-dashed lines) at $h/E_F = 0.5$. The upper and lower panels show the results with and without a magnetic impurity, respectively. The cross-patterns highlight the areas of topological superfluids.

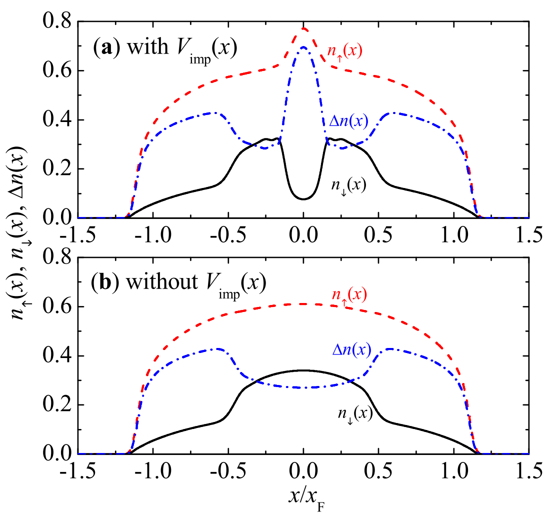

Figure 5: (color online) The spin-up and spin-down density distribution, $n_\uparrow$ (dashed lines) and $n_\downarrow$ (solid lines), and their difference $\Delta n = n_\uparrow - n_\downarrow$ (dot-dashed lines) at $h/E_F = 0.5$.

local density is enhanced and reduced for the spin-up and spin-down atoms, respectively, due to the magnetic impurity. This is consistent with the picture of inducing a local Zeeman field, as mentioned in the above.

Experimental proposal. - The model Hamiltonian Eq. (1) can be realized using ultracold ^{40}K atoms [14]. The two spin-1/2 states can be chosen as the two magnetic sublevels $|\uparrow\rangle = |F = 9/2, m_F = -9/2\rangle$ and $|\downarrow\rangle = |F = 9/2, m_F = -7/2\rangle$ of the $F = 9/2$ hyperfine level. To create the spin-orbit coupling, one can use a pair of counter-propagating Raman laser beams with strength Ω_R, which flip atoms from $|\downarrow\rangle$ to $|\uparrow\rangle$ spin states and simultaneously impart momentum $2\hbar k_R$ (or vice versa), via the two-photon Raman process [14]. This is the same technique used in the National Institute of Standards and Technology (NIST) to generate synthetic spin-orbit coupling in a Bose-Einstein condensate of ^{87}Rb atoms [19].

In our model Hamiltonian, the spin-orbit coupling strength is related to the wave-vector k_R by $\lambda = \hbar^2 k_R/(2m)$, and the effective Zeeman field can be tuned by the strength of Raman laser beam, i.e., $h = \Omega_R/2$. To make the system one-dimensional, one can impose 2D optical lattices that restrict atoms moving along a single tube. This was already demonstrated at Rice University to obtain an imbalanced 1D Fermi gas [16]. It is possible to create an impurity potential by using a laser beam that has a sufficient narrow beam width, as schematically shown in Fig. 6. By suitably tuning its frequency, the scattering potential caused by the laser beam can be attractive for the spin-up state and repulsive for the spin-down state. This localized probe therefore behaves like a classical magnetic impurity. All the techniques required to simulate the model Hamiltonian Eq. (1) are therefore within the reach of current experiments.

We note that our scheme does not yet lead to a fully developed quantum gate, which in any event requires a careful analysis of quantum dynamics during the switching process. Instead, it provides a simple demonstration system whereby the concepts of creating and moving Majorana fermions in a quasi one-dimensional environment - like ‘birds on a wire’ - can be usefully investigated.

Conclusions. - In summary, we have theoretically proposed how to manipulate Majorana fermions in a one-dimensional spin-orbit coupled atomic Fermi gas in a harmonic trap. The transportation of Majorana fermions through the Fermi cloud may be controlled by an effective Zeeman field, while the creation of pairs of Majorana fermions can be achieved through an artificial magnetic impurity created using a dimple laser beam. Our pro-

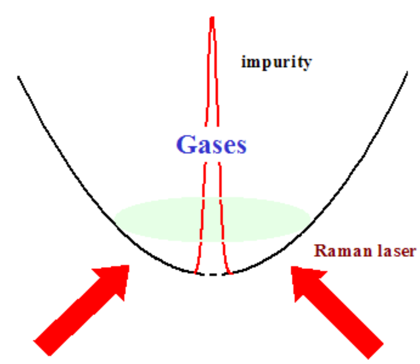

Figure 6: (color online) Schematic plot of the experimental setup. The spin-orbit coupling is induced by a pair of Raman laser beams, while the magnetic impurity with finite width is created by a dimple laser beam.

posal may be viewed as the cold-atom analog of the control gates proposed in the solid-state systems [6]. We anticipate that these control operations can be realized using ultracold ^{40}K atoms [14].

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